كـلـيــة هندسة الحاسوب والمعلوماتية والاتصالات
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Logic Circuits

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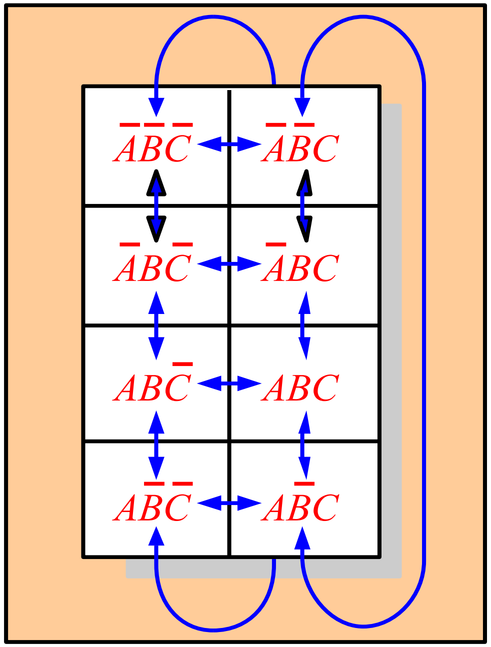
Chapter_ 4
Boolean Algebra

Lecture _07
Karnaugh Map
(خريطة كارنوف)

7-1. The Karnaugh Map

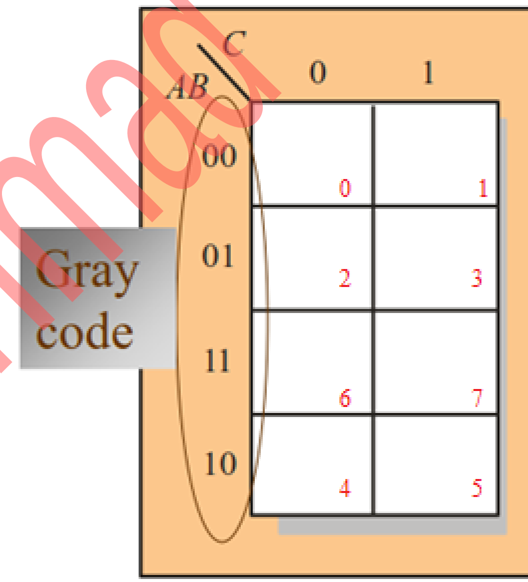
- The **Karnaugh map (K-map)** is similar to a truth table.
- The **K-map** is an array of **cells** (مصفوفة من الخلايا) in which each cell represent a binary value of the input variables.
- The **K-map** is a tool for simplifying combinational logic with two, three, four, and five variables.
- For 3 variables, 8 cells are required (2^3), and for 4 variables: 2^4=16 cells.

- The map shown is for three variables labeled *A*, *B*, and *C*.
  - Each cell represents one possible product term (مصطلح جدائي).
  - Each cell differs from an adjacent (المجاورة) cell by only one variable.



Karnaugh map

- Cells are usually labeled using 0's and 1's to represent the variable and its complement.
- The numbers are entered in **Gray code**, to force adjacent cells to be different by only one variable.
- **Ones** are read as the **true variable** and **zeros** are read as the **complemented variable**.

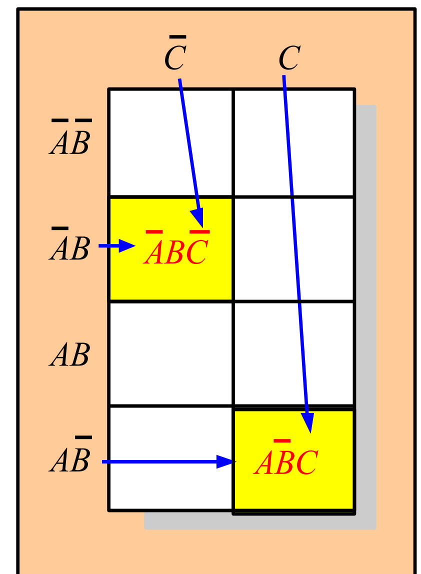


Example 7-1

Read the terms for the yellow cells.

Solution

The cells are $\overline{A}B\overline{C}$ and $A\overline{B}C$.



The 3-variable Karnaugh map

- The 3-variable K-maps is any array of eight cells, as shown in Fig. (a).
- Fig. (b) shows the standard product terms that are represented by each cell in the 3-variable K-map.

$AB \backslash C$	0	1
00	000	001
01	010	011
11	110	111
10	100	101

(a)

$AB \backslash C$	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

(b)

The 4-variable Karnaugh map

- The 4-variable K-maps is any array of sixteen cells, as shown in Fig. (a).
- Fig. (b) shows the standard product terms that are represented by each cell in the 4-variable K-map.

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

(a)

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

(b)

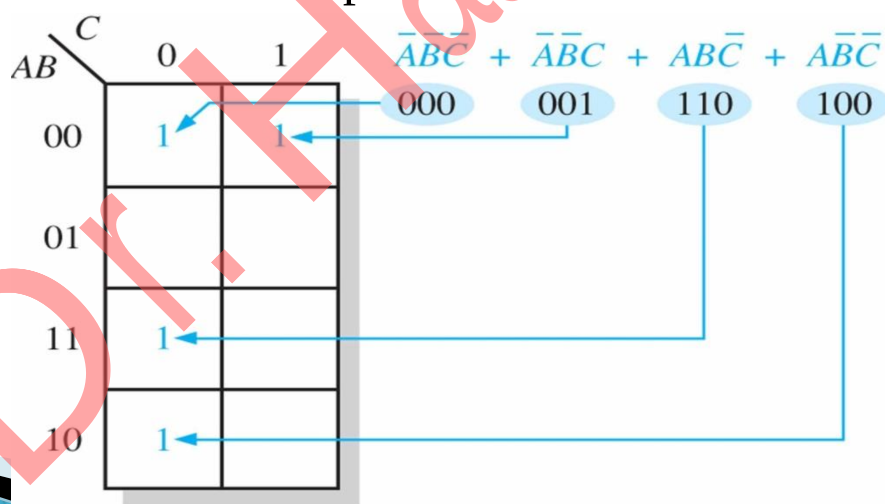
7-2. Karnaugh Map SOP Minimization

7-2-1. Mapping a standard SOP Expression

- For an SOP expression in standard form, a **1** is placed on the K-map for each product term in the expression.
  - **For example**, for the product term $A\overline{B}C$, a **1** goes in the 101 cell on a 3-variables map.
  - The cells that do not have a 1 are the cells for which the expression is **0**.
  - **The mapping process**:

Step 1. Determine the binary value of each product sum in the standard SOP expression.

Step 2. As each product term is evaluated, place a 1 on a K-map in the cell having the same value as the product term.



Example 7-2

Map the following standard SOP expression on a K-map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

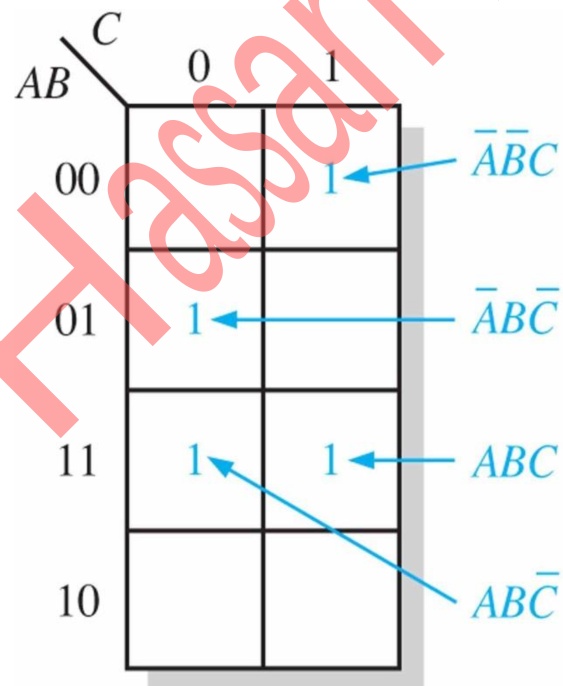
Solution

Evaluate the expression as show below.

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

0 0 1 0 1 0 1 1 0 1 1 1

Place a 1 on the 3-variable K-map in Fig. for each standard product term in the expression.



Example 7-3

Map the following standard SOP expression on a K-map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

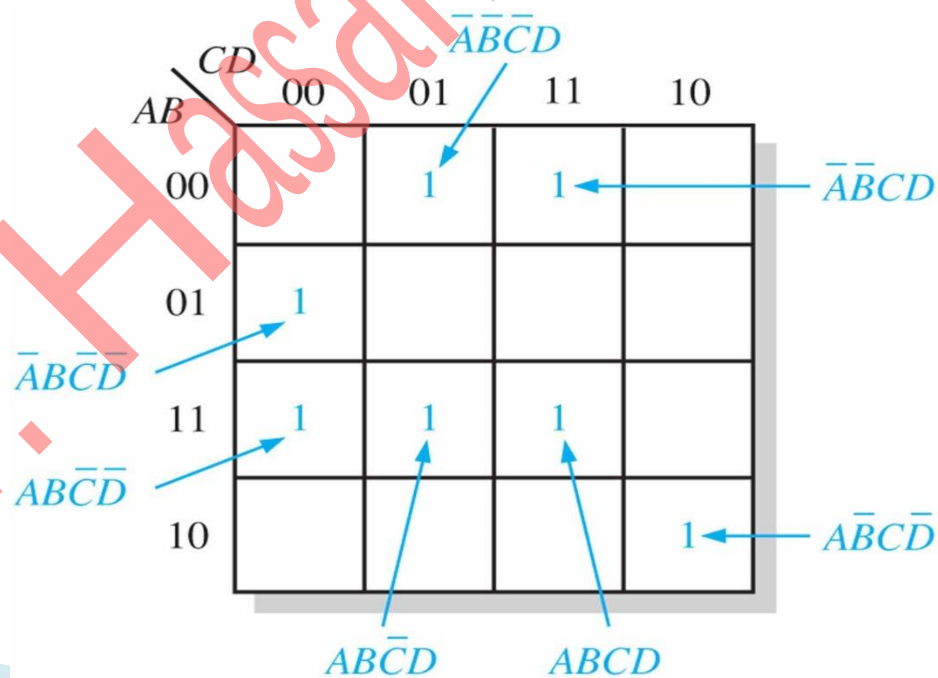
Solution

Evaluate the expression as show below.

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

0011 0100 1101 1111 1100 0001 1010

Place a 1 on the 4-variable K-map in Fig. for each standard product term in the expression.



7-2-2. Karnaugh Map Simplification of SOP Expression

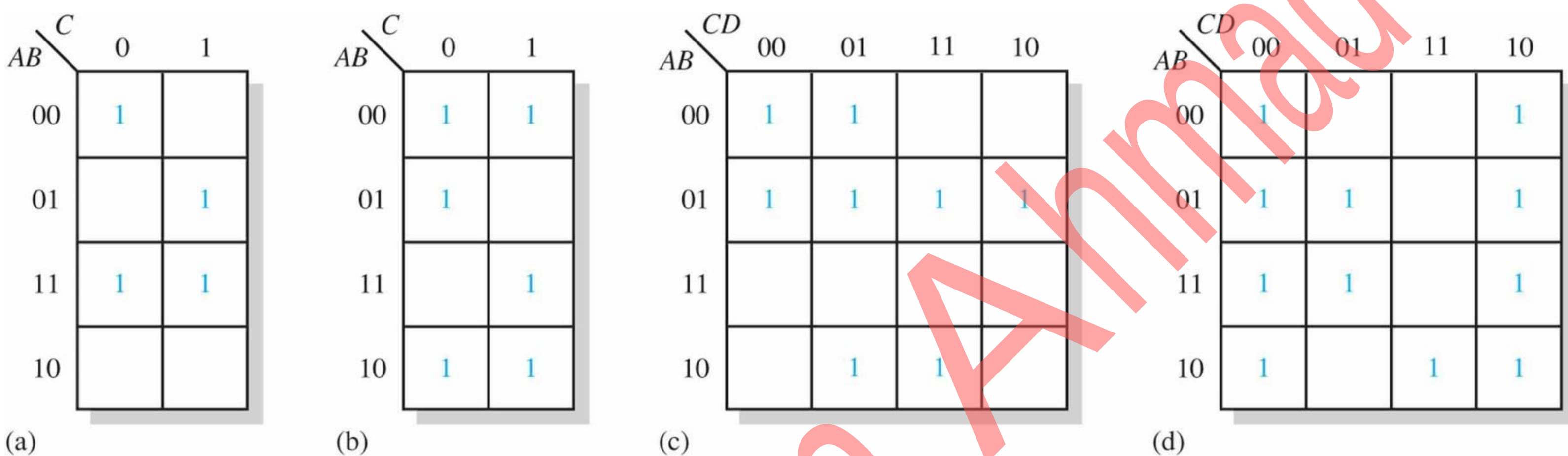
- After an SOP expression has been mapped, a minimum SOP expression is obtained by grouping the **1**s and determining the minimum SOP expression from map.

Grouping the 1s.

- You can group **1**s on the K-map by enclosing (إحاطة) those adjacent (المتجاورة) cells containing **1**s.
- Each group is either an square or rectangle shape.

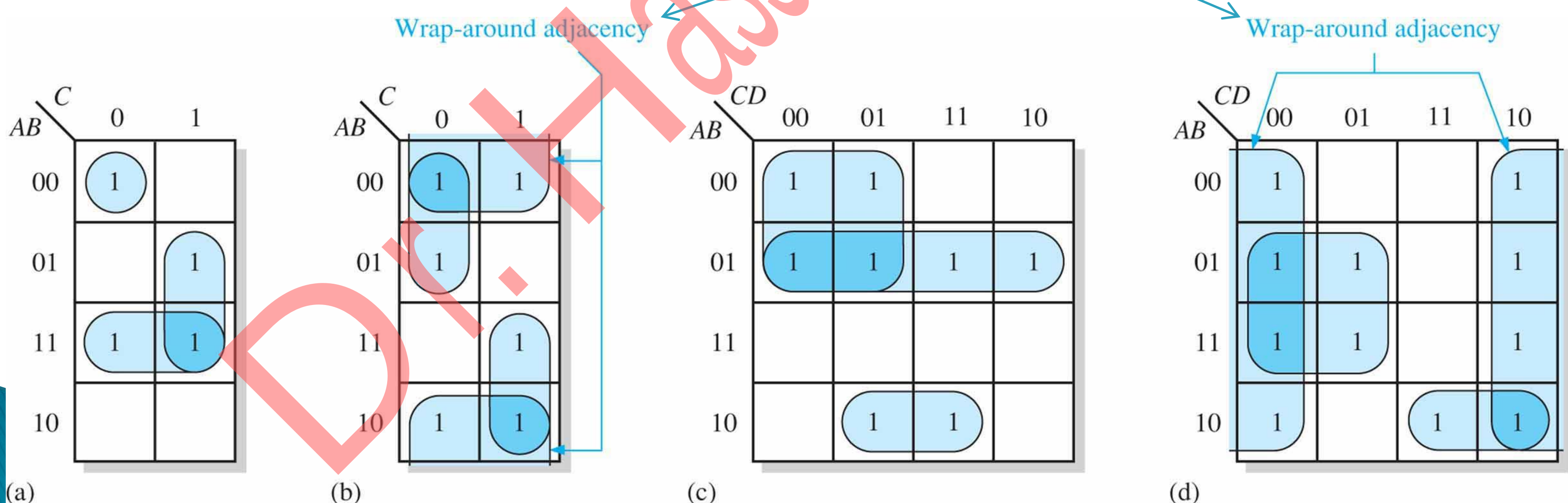
Example 7-4

Group 1s in each of the K-maps:



Solution

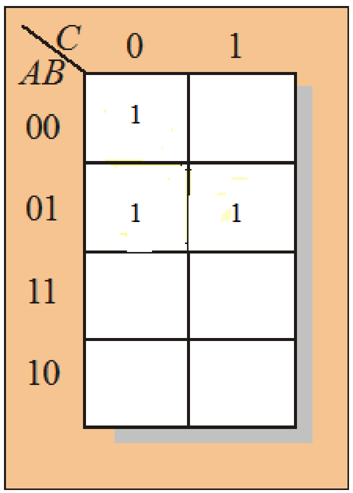
التفاف حول المتجاورة



- K-maps can **simplify** combinational logic by grouping cells and eliminating (إقصاء) variables that change.

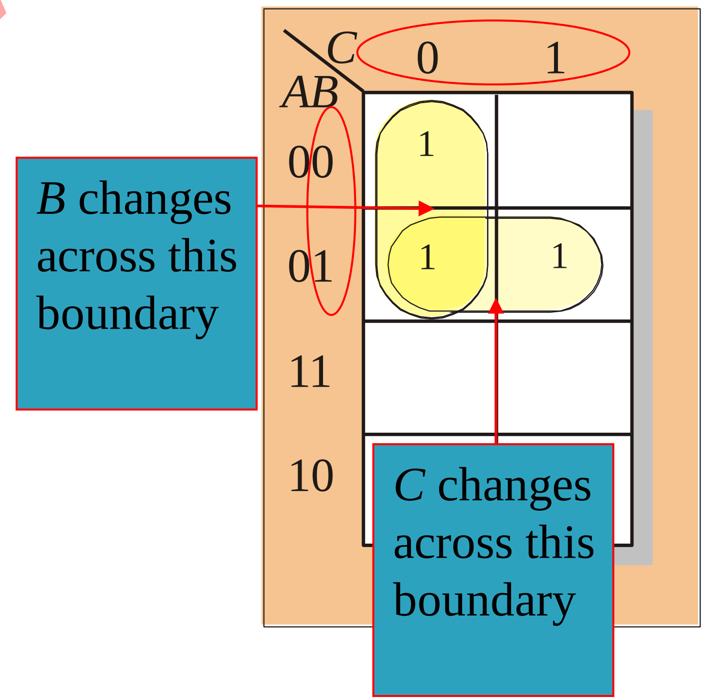
Example 7-5

Group the 1’s on the map and read the minimum logic.



Solution

1. Group the 1’s into two overlapping groups as indicated.
2. Read each group by eliminating (إزالة/إقصاء) any variable that changes across a boundary (الحد).
3. The vertical group is read $\overline{A}\,\overline{C}$.
4. The horizontal group is read $\overline{A}B$.



$$X = \overline{A}\,\overline{C} + \overline{A}B$$

- A 4-variable map has an adjacent cell on each of its four boundaries.

Example 7-6

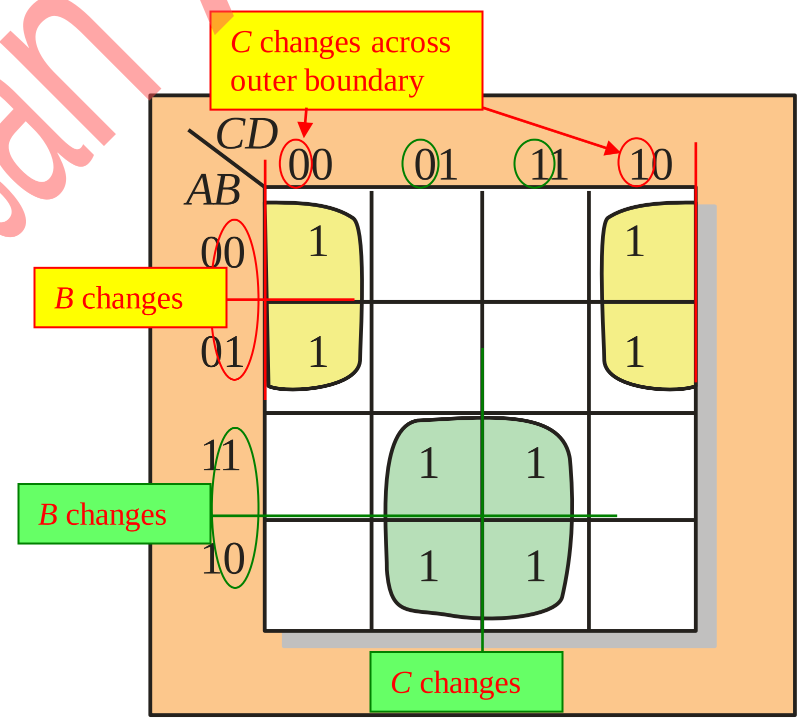
Group the 1's on the map and read the minimum logic.

Solution

1. Group the 1's into two separate groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The upper (yellow) group is read as $\overline{A}\,\overline{D}$.
4. The lower (green) group is read as AD.

AB \ CD	00	01	11	10
00	1			1
01	1			1
11		1	1	
10		1	1	

$$X = \overline{A}\,\overline{D} + AD$$



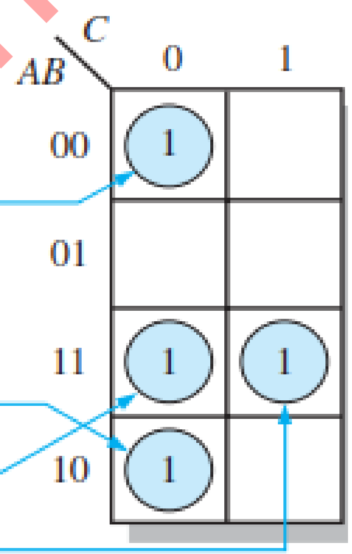
X

7-2-3. Mapping Directly from a Truth Table

- An example of a Boolean expression and its truth table representation is in Fig.
  - Notice in truth table that the output ***X*** is **1** for four different input variable combinations.
  - The **1**s in the output column of the truth table are mapped directly onto a K-map into the cells corresponding to the values of the associated input variable combinations, as shown in Fig.

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

Inputs			Output
A	**B**	**C**	**X**
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



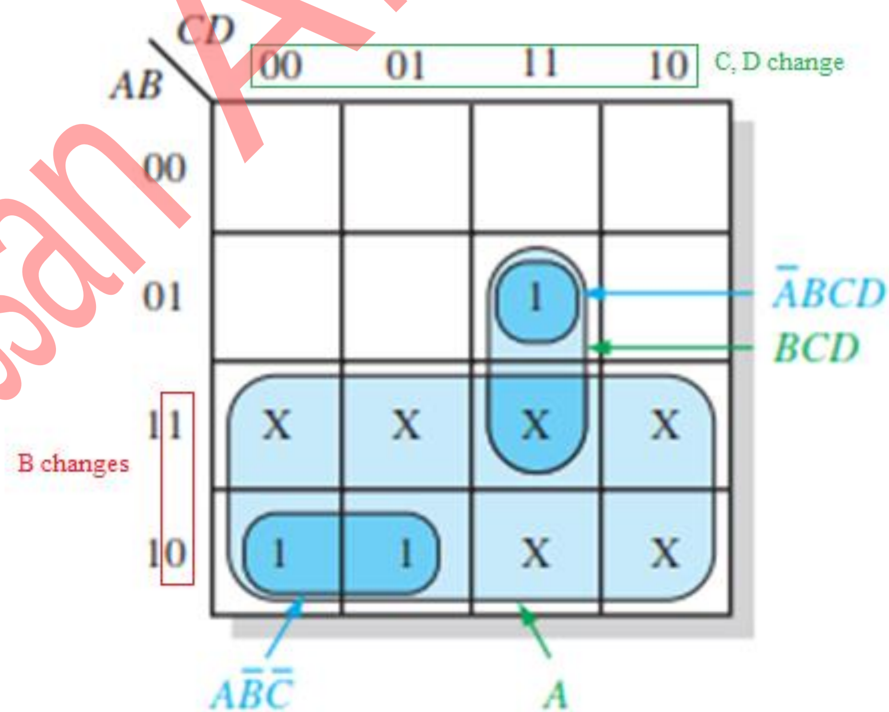
7-2-4. Don't Care Conditions

- Sometimes a situation arises in which some input variable combinations are not allowed.
- These combinations can be treated (يتم التعامل) as "don't care" (غير مهم / مهمل) terms with respect to (بالنسبة لـ) their effect on the output.
  - That is, for these "don't care" terms either a **1** or a **0** may be assigned to the output.
- For each "don't care" term, an **X** is placed in the cell.
- When grouping the **1**s, the **X**s can be treated as **1**s to make a larger grouping or as **0**s if they cannot be used to advantage (للاستفادة/زيادة المنفعة).
  - The larger a group, the simpler the resulting term will be.

- The truth table in Fig. (a) describes a logic function that has a 1 output only when the BCD code for 7, 8, or 9 is present on the inputs.
  - If the "don't cares" are used as 1s, the resulting expression for the function is $A + BCD$, as indicated in Fig.(b).
  - If the "don't cares" are not used as 1s, the resulting expression is $A\overline{B}\overline{C} + \overline{A}BCD$
  - So, this we can see the advantage of using "don't care" terms to get the simplest expression.

Inputs				Output
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

(a) Truth table



(b) Without "don't cares" $Y = A\overline{B}\overline{C} + \overline{A}BCD$
With "don't cares" $Y = A + BCD$

Selected Key Terms

Variable	A symbol used to represent a logical quantity that can have a value of 1 or 0, usually designated by an italic letter.
Complement	The inverse or opposite of a number. In Boolean algebra, the inverse function, expressed with a bar over the variable.
Sum term	The Boolean sum of two or more literals equivalent to an OR operation.
Product term	The Boolean product of two or more literals equivalent to an AND operation.
Sum-of-products (SOP)	A form of Boolean expression that is basically the ORing of ANDed terms.
Product of sums (POS)	An arrangement of cells representing combinations of literals in a Boolean expression and used for systematic simplification of the expression.
"Don't care"	A combination of input literals that cannot occur and can be used as a 1 or a 0 on a Karnaugh map for simplification.

True/False Quiz

1. *Variable, complement,* and *literal* are all terms used in Boolean algebra.
2. Addition in Boolean algebra is equivalent to the NOR function.
3. Multiplication in Boolean algebra is equivalent to the AND function.
4. The commutative law, associative law, and distributive law are all laws in Boolean algebra.
5. The complement of 0 is 0 itself.
6. When a Boolean variable is multiplied by its complement, the result is the variable.
7. "The complement of a product of variables is equal to the sum of the complements of each variable" is a statement of DeMorgan's theorem.
8. SOP means sum-of-products.
9. Karnaugh maps can be used to simplify Boolean expressions.
10. A 3-variable Karnaugh map has six cells.

1. T 2. F 3. T 4. T 5. F 6. F 7. T 8. T 9. T 10. F

SELF-TEST-1

1. The associative law for addition is normally written as
   - a. $A + B = B + A$
   - b. $\mathbf{(A + B) + C = A + (B + C)}$
   - c. $AB = BA$
   - d. $A + AB = A$
2. The Boolean equation $AB + AC = A(B+ C)$ illustrates
   - a. the distribution law
   - b. the commutative law
   - c. **the associative law**
   - d. DeMorgan's theorem
3. The Boolean expression $A \cdot 1$ is equal to
   - a. $\mathbf{A}$
   - b. B
   - c. 0
   - d. 1
4. The Boolean expression $A + 1$ is equal to
   - a. A
   - b. B
   - c. 0
   - d. **1**

5. The Boolean equation $AB + AC = A(B + C)$ illustrates
   a. **the distribution law**
   b. the commutative law
   c. the associative law
   d. DeMorgan's theorem
6. A Boolean expression that is in standard SOP form is
   a. the minimum logic expression
   b. contains only one product term
   c. **has every variable in the domain in every term**
   d. none of the above
7. Adjacent cells on a Karnaugh map differ from each other by
   a. **one variable**
   b. two variables
   c. three variables
   d. answer depends on the size of the map

8. The minimum expression that can be read from the Karnaugh map shown is

   a. ***X* = *A***
   b. $X = A$
   c. $X = B$
   d. $X = B$

	$\bar{C}$	C
$\bar{A}\bar{B}$		
$\bar{A}B$		
AB	1	1
$A\bar{B}$	1	1

9. The minimum expression that can be read from the Karnaugh map shown is

   a. $X = A$
   b. $X = A$
   c. $X = B$
   d. $X = B$

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$		
AB		
$A\bar{B}$	1	1

SELF-TEST-2

1. A variable is a symbol in Boolean algebra used to represent
   (a) data
   (b) a condition
   (c) an action
   (d) answers (a), (b), and (c)
2. The Boolean expression $A + B + C$ is
   (a) a sum term
   (b) a literal term
   (c) an inverse term
   (d) a product term
3. The Boolean expression $\overline{ABCD}$ is
   (a) a sum term
   (b) a literal term
   (c) an inverse term
   (d) a product term
4. The domain of the expression $A\overline{B}CD + A\overline{B} + \overline{C}D + B$ is
   (a) A and D
   (b) B only
   (c) A, B, C, and D
   (d) none of these
5. According to the associative law of addition,
   (a) $A + B = B + A$
   (b) $A = A + A$
   (c) $(A + B) + C = A + (B + C)$
   (d) $A + 0 = A$
6. According to commutative law of multiplication,
   (a) $AB = BA$
   (b) $A = AA$
   (c) $(AB)C = A(BC)$
   (d) $A0 = A$
7. According to the distributive law,
   (a) $A(B + C) = AB + AC$
   (b) $A(BC) = ABC$
   (c) $A(A + 1) = A$
   (d) $A + AB = A$

1. (d) **2.** (a) **3.** (d) **4.** (c) **5.** (c) **6.** (a) **7.** (a)

8. Which one of the following is *not* a valid rule of Boolean algebra?
   (a) $A + 1 = 1$ (b) $A = \overline{A}$
   (c) $AA = A$ (d) $A + 0 = A$

9. Which of the following rules states that if one input of an AND gate is always 1, the output is equal to the other input?
   (a) $A + 1 = 1$ (b) $A + A = A$
   (c) $A \cdot A = A$ (d) $A \cdot 1 = A$

10. According to DeMorgan's theorems, the complement of a product of variables is equal to
    (a) the complement of the sum (b) the sum of the complements
    (c) the product of the complements (d) answers (a), (b), and (c)

11. The Boolean expression $X = (A + B)(C + D)$ represents
    (a) two ORs ANDed together (b) two ANDs ORed together
    (c) A 4-input AND gate (d) a 4-input OR gate

12. An example of a sum-of-products expression is
    (a) $A + B(C + D)$ (b) $\overline{A}B + A\overline{C} + A\overline{B}C$
    (c) $(\overline{A} + B + C)(A + \overline{B} + C)$ (d) both answers (a) and (b)

13. An example of a product-of-sums expression is
    (a) $A(B + C) + A\overline{C}$ (b) $(A + B)(\overline{A} + B + \overline{C})$
    (c) $\overline{A} + \overline{B} + BC$ (d) both answers (a) and (b)

14. An example of a standard SOP expression is
    (a) $\overline{A}B + A\overline{B}C + AB\overline{D}$ (b) $A\overline{B}C + A\overline{C}D$
    (c) $A\overline{B} + \overline{A}B + AB$ (d) $A\overline{B}C\overline{D} + \overline{A}B + \overline{A}$

8. (b) **9.** (d) **10.** (b) **11.** (a) **12.** (b) **13.** (b) **14.** (c)

Problems & Solutions

Prob. 4-1

Find the values of the variables that make each product term 1 and each sum term 0.

(a) AB (b) $A\overline{B}C$ (c) $A + B$ (d) $\overline{A} + B + \overline{C}$

(e) $\overline{A} + \overline{B} + C$ (f) $\overline{A} + B$ (g) $A\overline{B}\,\overline{C}$

Sol.

(a) $AB = 1$ when $A = 1$, $B = 1$

(b) $A\overline{B}C = 1$ when $A = 1$, $B = 0$, $C = 1$

(c) $A + B = 0$ when $A = 0$, $B = 0$

(d) $\overline{A} + B + \overline{C} = 0$ when $A = 1$, $B = 0$, $C = 1$

(e) $\overline{A} + \overline{B} + C = 0$ when $A = 1$, $B = 1$, $C = 0$

(f) $\overline{A} + B = 0$ when $A = 1$, $B = 0$

(g) $A\overline{B}\,\overline{C} = 1$ when $A = 1$, $B = 0$, $C = 0$

Prob. 4-2

Find the value of ***X*** for all possible values of the variables.

(a) $X = (A + B)C + B$ **(b)** $X = (\overline{A + B})C$ **(c)** $X = A\overline{B}C + AB$

(d) $X = (A + B)(\overline{A} + B)$ **(e)** $X = (A + BC)(\overline{B} + \overline{C})$

Sol.

(a) $X = (A + B)C + B$

A	B	C	$A + B$	$(A + B)C$	X
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1

(b) $X = (\overline{A+B})C$

A	B	C	$\overline{A+B}$	X
0	0	0	1	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

(c) $X = A\overline{B}C + AB$

A	B	C	$A\overline{B}C$	AB	X
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	0	1	1

(d) $X = (A + B)(\overline{A} + B)$

A	B	$A + B$	$\overline{A}+B$	X
0	0	0	1	0
0	1	1	1	1
1	0	1	0	0
1	1	1	1	1

(e) $X = (A + BC)(\overline{B}+\overline{C})$

A	B	C	$A + BC$	$\overline{B}+\overline{C}$	X
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	0	0

Prob. 4-3

Apply DeMorgan's theorems to each expression:

(a) $\overline{A+\overline{B}}$ (b) $\overline{\overline{A}B}$ (c) $\overline{A+B+C}$ (d) $\overline{ABC}$

(e) $\overline{A(B+C)}$ (f) $\overline{AB}+\overline{CD}$ (g) $\overline{AB+CD}$ (h) $\overline{(A+\overline{B})(\overline{C}+D)}$

Sol.

(a) $\overline{A+\overline{B}} = \overline{A}\overline{\overline{B}} = \overline{\boldsymbol{A}}\boldsymbol{B}$

(b) $\overline{\overline{A}B} = \overline{\overline{A}} + \overline{B} = \boldsymbol{A} + \overline{\boldsymbol{B}}$

(c) $\overline{A+B+C} = \overline{\boldsymbol{A}}\,\overline{\boldsymbol{B}}\,\overline{\boldsymbol{C}}$

(d) $\overline{ABC} = \overline{\boldsymbol{A}} + \overline{\boldsymbol{B}} + \overline{\boldsymbol{C}}$

(e) $\overline{A(B+C)} = \overline{A} + \overline{(B+C)} = \overline{\boldsymbol{A}} + \overline{\boldsymbol{B}}\,\overline{\boldsymbol{C}}$

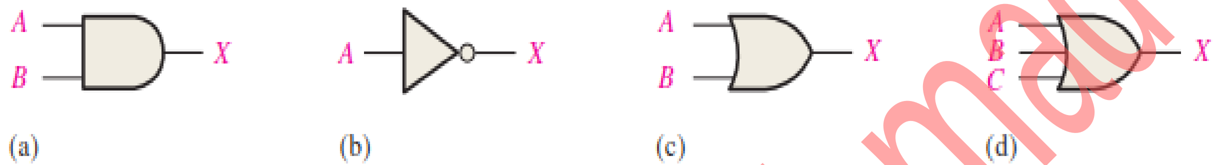
(f) $\overline{AB}+\overline{CD} = \overline{\boldsymbol{A}} + \overline{\boldsymbol{B}} + \overline{\boldsymbol{C}} + \overline{\boldsymbol{D}}$

(g) $\overline{AB+CD} = (\overline{AB})(\overline{CD}) = (\overline{\boldsymbol{A}} + \overline{\boldsymbol{B}})(\overline{\boldsymbol{C}} + \overline{\boldsymbol{D}})$

(h) $\overline{(A+\overline{B})(\overline{C}+D)} = \overline{A+\overline{B}} + \overline{\overline{C}+D} = \overline{\boldsymbol{A}}\boldsymbol{B} + \boldsymbol{C}\overline{\boldsymbol{D}}$

Prob. 4-4

Write the Boolean expression for each of the logic gates in Fig.



Sol.

(a) $AB = X$

(b) $\overline{A} = X$

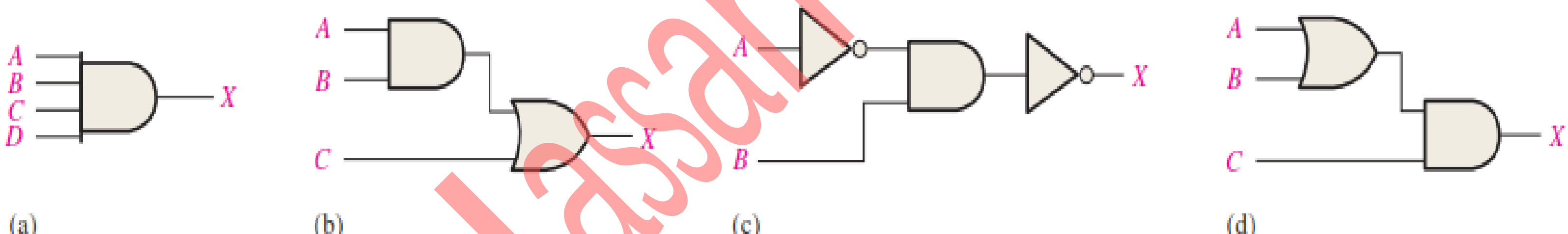
(c) $A + B = X$

(d) $A + B + C = X$

****

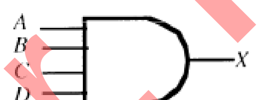
Prob. 4-5

Write the Boolean expression for each of the logic circuits in Fig.

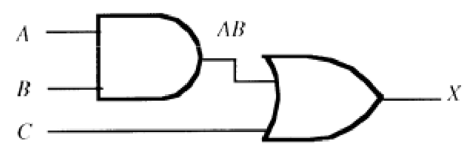


Sol.

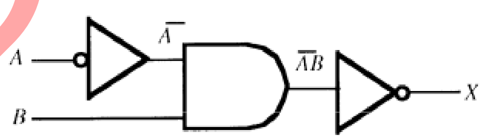
(a) $X = ABCD$



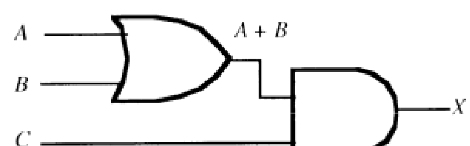
(b) $X = AB + C$



(c) $X = \overline{\overline{A}B}$



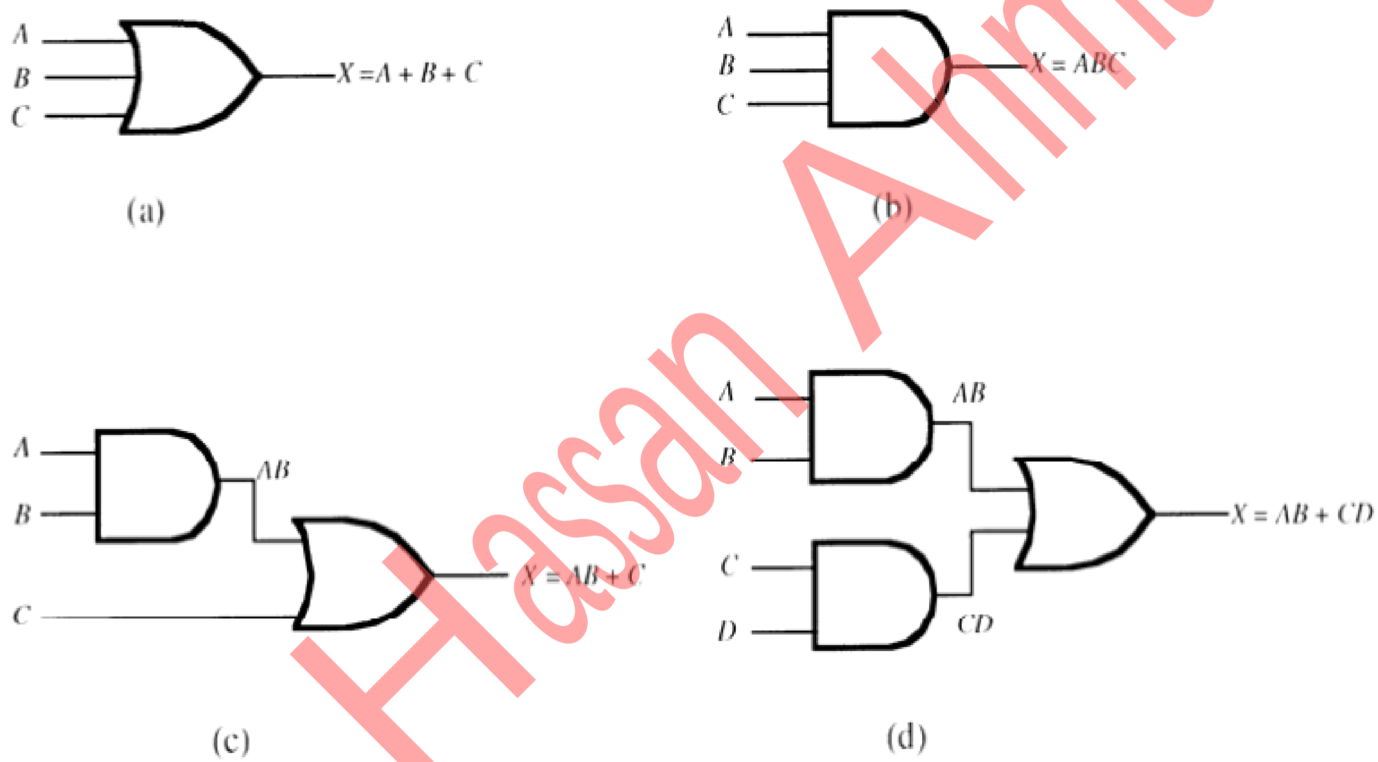
(d) $X = (A + B)C$



Prob. 4-6 Draw the logic circuit represented by each of the following expressions:

(a) $A + B + C$ (b) ABC (c) $AB + C$ (d) $AB + CD$

Sol.



Prob. 4-7 Construct a truth table for each of the following Boolean expressions:

(a) $A + B$ (b) AB (c) $AB + BC$

(d) $(A + B)C$ (e) $(A + B)(\overline{B} + C)$

Sol.

(a) $A + B$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(b) AB

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

(c) $X = AB + BC$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(d) $X = (A + B)C$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(e) $X = (A+B)(\overline{B}+C)$

A	B	C	$A + B$	$\overline{B}+C$	X
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Prob. 4-8

Using Boolean algebra, simplify the following expressions:

(a) $BD + B(D + E) + \bar{D}(D + F)$

(b) $\bar{A}\bar{B}C + \overline{(A + B + \bar{C})} + \bar{A}\bar{B}\bar{C}D$

(c) $(B + BC)(B + \bar{B}C)(B + D)$

(d) $ABCD + AB(\overline{CD}) + (\overline{AB})CD$

(e) $ABC[AB + \bar{C}(BC + AC)]$

Sol.

(a) $BD + B(D+E) + \bar{D}(D+F) = BD + BD + BE + \bar{D}D + \bar{D}F$

$= BD + BE + 0 + \bar{D}F = \mathbf{BD + BE + \bar{D}F}$

(b) $\bar{A}\bar{B}C + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D = \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}(C + \bar{C}D) = \bar{A}\bar{B}(C + D) = \mathbf{\bar{A}\bar{B}C + \bar{A}\bar{B}D}$

(c) $(B + BC)(B + \bar{B}C)(B + D) = B(1 + C)(B + C)(B + D)$

$= B(B+C)(B+D) = (BB + BC)(B+D) = (B + BC)(B + D)$

$= B(1 + C)(B + D) = B(B + D) = BB + BD = B + BD = B(1 + D) = \mathbf{B}$

(d) $ABCD + AB(\overline{CD}) + (\overline{AB})CD = ABCD + AB(\bar{C} + \bar{D}) + (\bar{A} + \bar{B})CD$

$= ABCD + AB\bar{C} + AB\bar{D} + \bar{A}CD + \bar{B}CD$

$= CD(AB + \bar{A} + \bar{B}) + AB\bar{C} + AB\bar{D} = CD(B + \bar{A} + \bar{B}) + AB\bar{C} + AB\bar{D}$

$= CD(1 + \bar{A}) + AB\bar{C} + AB\bar{D} = CD + AB\bar{C} + AB\bar{D} = CD + AB(\overline{CD}) = \mathbf{CD + AB}$

(e) $ABC[AB + \bar{C}(BC + AC)] = ABABC + ABC\bar{C}(BC + AC)$

$= ABC + 0(BC + AC) = \mathbf{ABC}$

Prob. 4-9

Convert the following expressions to sum-of-product (SOP) forms:

(a) $AB + CD(A\overline{B} + CD)$ **(b)** $AB(\overline{B}\,\overline{C} + BD)$ **(c)** $A + B[AC + (B + \overline{C})D]$

Sol.

(a) $AB + CD(A\overline{B} + CD) = AB + A\overline{B}CD + CDCD = AB + A\overline{B}CD + CD$

$= AB(A\overline{B} + 1)CD = \mathbf{AB + CD}$

(b) $AB(\overline{B}\,\overline{C} + BD) = AB\overline{B}\,\overline{C} + ABBD = 0 + ABD$ $=$ **ABD**

(c) $A + B[AC + (B + \overline{C})D] = A + ABC + (B + \overline{C})BD$

$= A + ABC + BD + B\overline{C}D = A(1 + BC) + BD + B\overline{C}D = A + BD(1 + \overline{C})$

$=$ **A + BD**

Prob. 4-10

Develop a truth table for each of the SOP expressions:

(a) $\overline{A}B + AB\overline{C} + \overline{A}\,\overline{C} + A\overline{B}C$ **(b)** $\overline{X} + Y\overline{Z} + WZ + X\overline{Y}Z$

Sol.

(a) $$\overline{A}B + AB\overline{C} + \overline{A}\,\overline{C} + A\overline{B}C = \overline{A}BC + \overline{A}B\overline{C} + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$$

(b) $$\begin{aligned}\overline{X} + Y\overline{Z} + WZ + X\overline{Y}Z = {} & \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z} + \overline{W}\,\overline{X}\,\overline{Y}Z + \overline{W}\,\overline{X}Y\overline{Z} + \overline{W}\,\overline{X}YZ \\ & + \overline{W}X\overline{Y}Z + \overline{W}XY\overline{Z} + W\overline{X}\,\overline{Y}\,\overline{Z} + W\overline{X}\,\overline{Y}Z \\ & + W\overline{X}Y\overline{Z} + W\overline{X}YZ + WX\overline{Y}Z + WXY\overline{Z} + WXYZ\end{aligned}$$

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

W	X	Y	Z	Q
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Prob. 4-11 Develop a truth table for each of the standard POS expressions:

(a) $(A + B)(A + C)(A + B + C)$

(b) $(A + \overline{B})(A + \overline{B} + \overline{C})(B + C + \overline{D})(\overline{A} + B + \overline{C} + D)$

Sol.

(a)

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(b)

A	B	C	D	X
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Prob. 4-12 For each truth table, derive a standard SOP and a standard POS expression.

Sol.

(a)

ABC	X
000	0
001	1
010	0
011	0
100	1
101	1
110	0
111	1

(b)

ABC	X
000	0
001	0
010	0
011	0
100	0
101	1
110	1
111	1

(c)

$ABCD$	X
0000	1
0001	1
0010	0
0011	1
0100	0
0101	1
0110	1
0111	0
1000	0
1001	1
1010	0
1011	0
1100	1
1101	0
1110	0
1111	0

(d)

$ABCD$	X
0000	0
0001	0
0010	1
0011	0
0100	1
0101	1
0110	0
0111	1
1000	0
1001	0
1010	0
1011	1
1100	1
1101	0
1110	0
1111	1

(a) $X = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + ABC$

$X = (A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+C)$

(b) $X = AB\overline{C} + A\overline{B}C + ABC$

$X = (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+C)$

(c) $X = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + AB\overline{C}\,\overline{D}$

$X = (A+B+\overline{C}+D)(A+\overline{B}+C+D)(A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+B+C+D)(\overline{A}+B+\overline{C}+D)$
$(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

Prob. 4-13 Draw a 3-variable Karnaugh map and label each cell according to its binary value.

Sol.

AB \ C	0	1
00	000	001
01	010	011
11	110	111
10	100	101

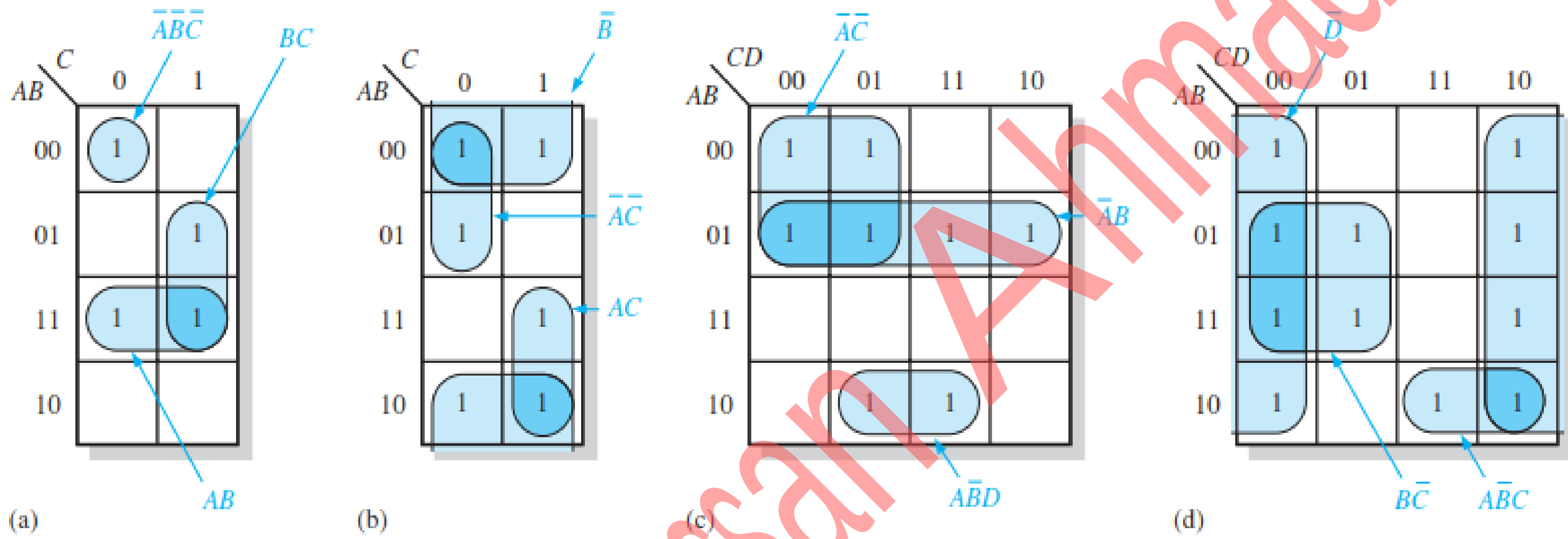
****

Prob. 4-14 Write the standard product term for each cell in a 3-variable Karnaugh map.

Sol.

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

Prob. 4-15 Given the Karnaugh maps in Figure. Write the resulting minimum SOP expression for each.



Sol. The resulting minimum product term for each group is shown in Figure. The minimum SOP expressions for each of the Karnaugh maps in the figure are

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$

(b) $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$

(d) $\overline{D} + A\overline{B}C + B\overline{C}$

Prob. 4-16 Expand each expression to a standard SOP form:

(a) $AB + A\bar{B}C + ABC$ **(b)** $A + BC$

(c) $A\bar{B}\bar{C}D + AC\bar{D} + B\bar{C}D + \bar{A}BC\bar{D}$ **(d)** $A\bar{B} + A\bar{B}\bar{C}D + C\bar{D} + B\bar{C}D + ABCD$

Sol.

(a)
$$AB + A\bar{B}C + ABC = AB(C+\bar{C}) + A\bar{B}C + ABC$$
$$= ABC + AB\bar{C} + A\bar{B}C + ABC$$
$$= ABC + A\bar{B}C + AB\bar{C}$$

(b)
$$A + BC = A(B+\bar{B})(C+\bar{C}) + (\bar{A}+A)BC = (AB + A\bar{B})(C+\bar{C}) + (\bar{A}+A)BC$$
$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + ABC$$
$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

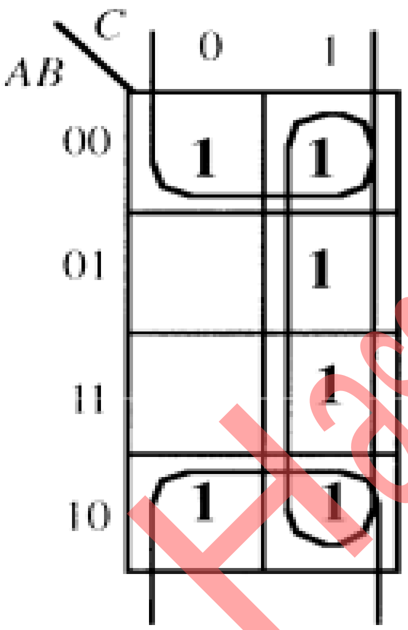
(c)
$$A\bar{B}\bar{C}D + AC\bar{D} + B\bar{C}D + \bar{A}BC\bar{D}$$
$$= A\bar{B}\bar{C}D + A(B+\bar{B})C\bar{D} + (A+\bar{A})B\bar{C}D + \bar{A}BC\bar{D} =$$
$$= A\bar{B}\bar{C}D + ABC\bar{D} + A\bar{B}C\bar{D} = AB\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BC\bar{D}$$

(d)
$$A\bar{B} + A\bar{B}\bar{C}D + C\bar{D} + B\bar{C}D + ABCD$$
$$= A\bar{B}(C+\bar{C})(D+\bar{D}) + A\bar{B}\bar{C}D + (A+\bar{A})(B+\bar{B})C\bar{D} + (A+\bar{A})B\bar{C}D + ABCD$$
$$= A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABC\bar{D} + ABCD + A\bar{B}\bar{C}D + ABCD + A\bar{B}CD + \bar{A}BCD$$
$$+ \bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}B\bar{C}D + ABCD$$
$$= A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABC\bar{D} + ABCD + A\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}CD + AB\bar{C}D + \bar{A}B\bar{C}D$$
$$= \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}CD + AB\bar{C}D + ABCD + ABC\bar{D}$$

Prob. 4-17 Reduce the function specified in truth Table to its minimum SOP form by using a Karnaugh map.

Inputs			Output
A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Sol. Plot the 1's from table in the text on the map as shown in Fig. and simplify.

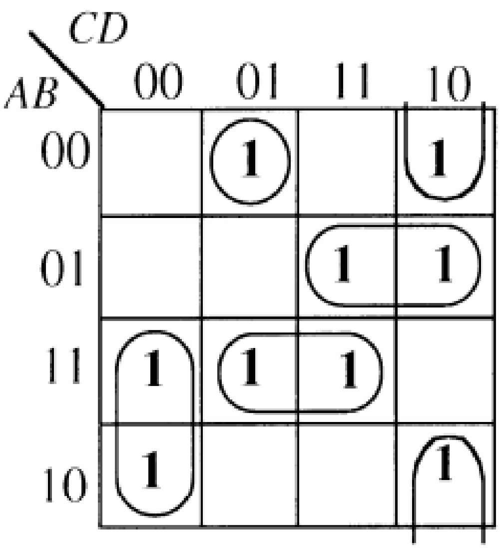


$X = \overline{B} + C$

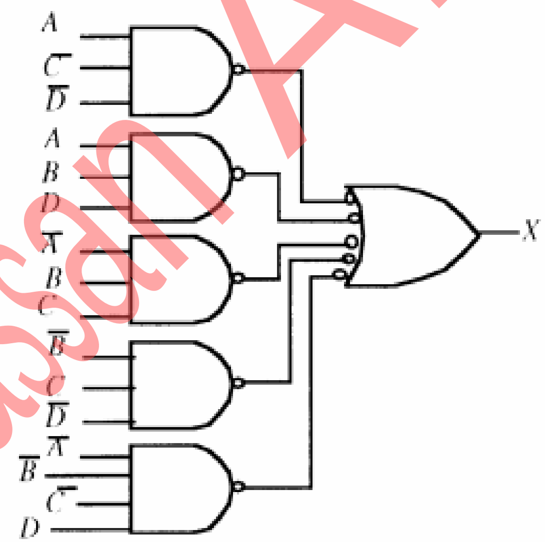
Prob. 4-18 Use the Karnaugh map method to implement the minimum SOP expression for the logic function specified in truth Table.

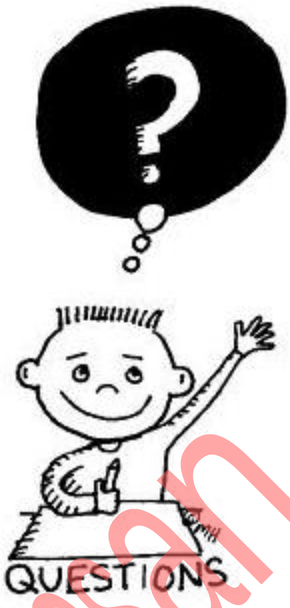
Inputs				Output
A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Sol.



$$X = A\overline{C}\,\overline{D} + ABD + \overline{A}BC + \overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D$$





The end of Lecture_07, chapter 4